# Multi-armed Bandits on the Web

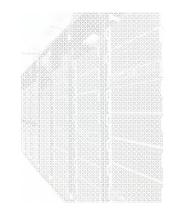
Successes, Lessons and Challenges

# Lihong Li

Microsoft Research 08/24/2014

2<sup>nd</sup> Workshop on User Engagement Optimization (KDD'14)

### BIG DATA



### correlation

Statistics, NL, DM, ...

# Big Trap

### KNOWLEDGE

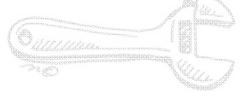


# **Correlation** ≠ Causation







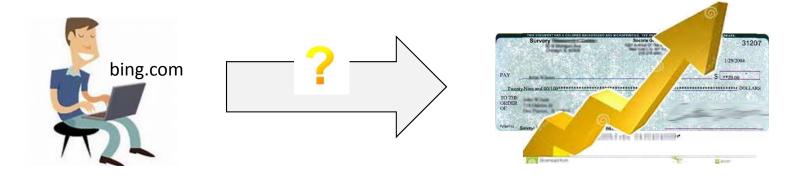


ACTION

# Somewhat Toy-ish Example

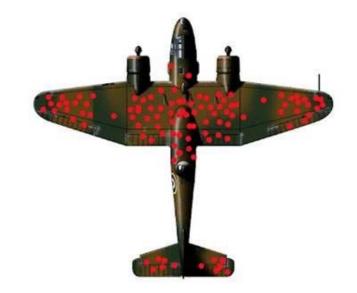
 Studies show... people who search their names in search engines tend to have higher income

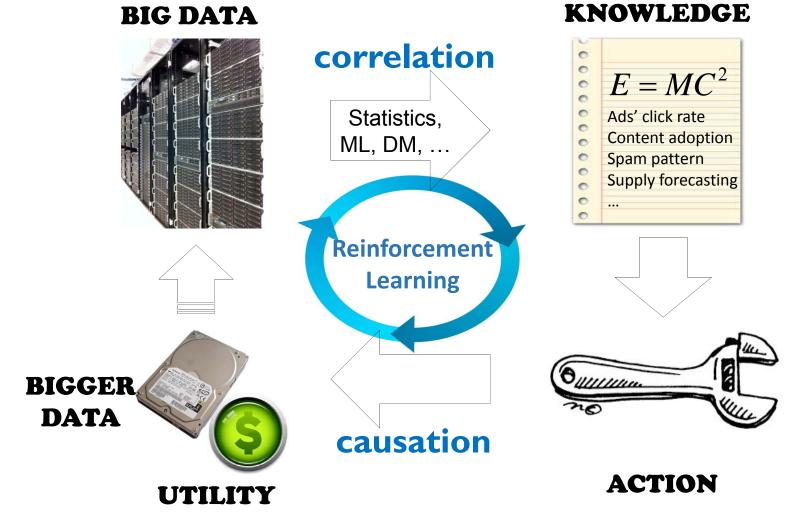
Decision making:



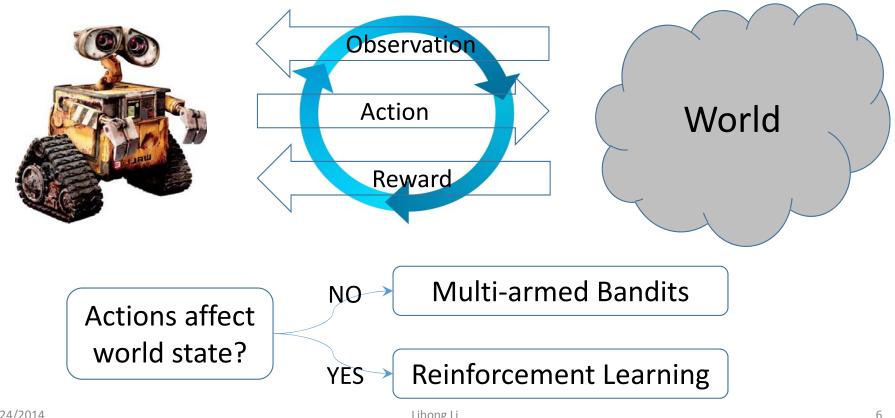
# WWII Example

- Statistics collected during WWII...
  - Bullet holes on bomber planes that came back from mission
- Decision making:
  - Where to armor?
  - Abraham Wald: the opposite!





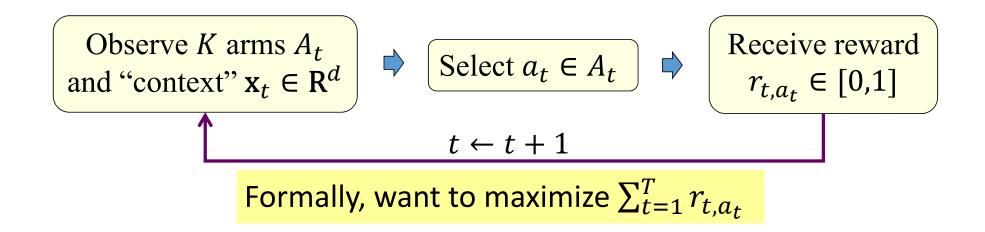
# Machine Learning for Decision Making



# Outline

- Multi-armed Bandits Algorithms
- Offline Evaluation
- Concluding Remarks

Contextual Bandit [Barto & co'85, Langford & co'08]



Generalizes classic K-armed bandits (without context)
Stochastic vs. adversarial

# Motivating Applications

- Clinical trials
- Resource allocation
- Queuing & scheduling
- ...
- Web (more recently)
  - Recommendation
  - Advertising
  - Search

# Case 1: Personalized News Recommendation

### www.yahoo.com



 $A_t$ : available articles at time t $\mathbf{x}_t$ : user features (age, gender, interests, ...)  $a_t$ : the displayed article at time t

 $r_{t,a}$ : 1 for click, 0 for no - click

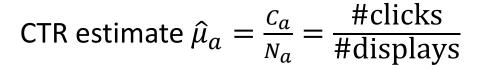
Average reward is click-through rate (CTR)

# Standard Multi-armed Bandit [R'52, LR'85]

No contextual information is available  $\rightarrow$  potentially lower rewards



 $CTR_1$ 





CTR<sub>2</sub>

 $\epsilon$ -greedy:

Choose article  $\begin{cases} \arg\max \hat{\mu}_a \text{, with prob. } 1 - \epsilon \\ \operatorname{random, with prob. } \epsilon \end{cases}$ 



CTR<sub>3</sub>

UCB1 (Upper Confidence Bound) [ACF'02] Choose article  $\arg\max_{a} \left\{ \hat{\mu}_{a} + \frac{\alpha}{\sqrt{N_{a}}} \right\}$ 

Exploration bonus that decays over time

# LinUCB: UCB for Linear Models [LCLS'10]

- Linear model assumption:  $\mathbf{E}[r_{t,a}|\mathbf{x}_t] = \mathbf{x}_t^\mathsf{T}\theta_a$
- Standard least-squares ridge regression

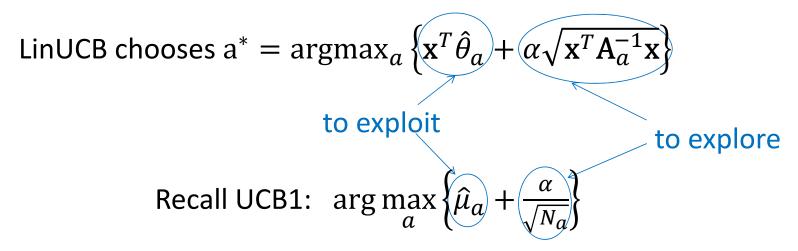
$$\hat{\theta}_a = (\mathbf{D}_a^T \mathbf{D}_a + \mathbf{I})^{-1} \mathbf{D}_a^T \mathbf{c}_a, \text{ where } \mathbf{D}_a = \begin{bmatrix} -\mathbf{x}_{t_1}^T - \\ -\mathbf{x}_{t_2}^T - \\ \vdots \end{bmatrix} \text{ and } \mathbf{c}_a = \begin{bmatrix} r_{t_1} \\ r_{t_2} \\ \vdots \end{bmatrix}$$

Quantifying prediction uncertainty: with high probability,

$$\left| \mathbf{x}^T \hat{\theta}_a - \mathbf{x}^T \theta_a \right| \le \alpha \sqrt{\mathbf{x}^T \mathbf{A}_a^{-1} \mathbf{x}}$$
Prediction error

Measures how similar x is to previous contexts

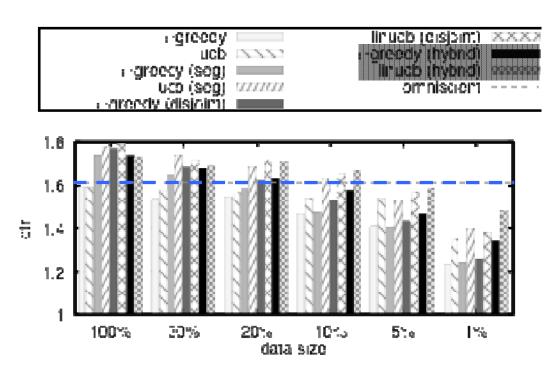
# LinUCB: Optimism in the Face of Uncertainty



A variant of LinUCB:  $O(\sqrt{KdT})$  with matching lower bound [CLRS'11]

LinRel [Auer 2002] is similarly motivated but more complicated.

# LinUCB for News Recommendation [LCLS'10]



- UCB-type algorithms do better than  $\varepsilon$ -greedy counterparts
- CTR improved significantly when features/contexts are considered

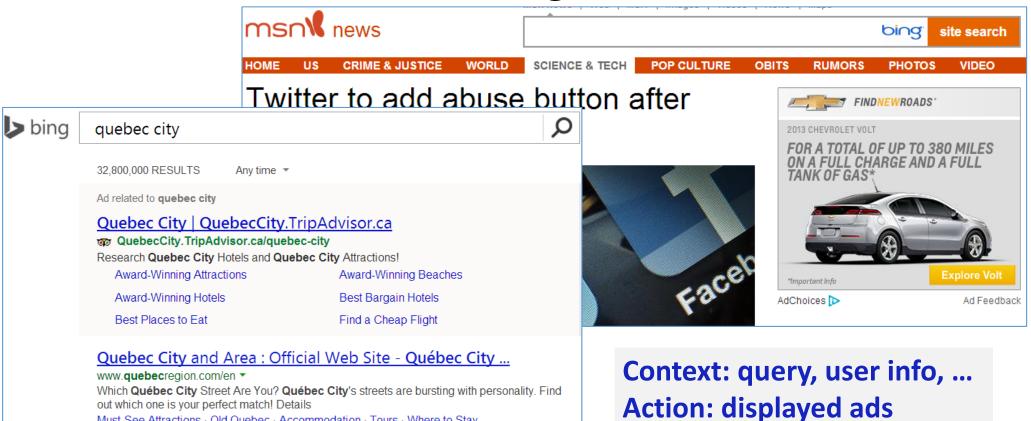
### LinUCB Variants

- Hybrid linear models for multi-task learning [<u>LCLS'10</u>]
  - Beneficial when data is sparse
- Generalized linear model [LCLMW'12]
  - Greater flexibility of modeling rewards
  - Linear regression, logistic regression, probit regression, ...
- Sparse linear model [Abbasi-Yadkori & co'12]
  - Lower regret when  $\{\theta_a\}$  are sparse

# Case 2: Online Advertising

Must-See Attractions · Old Quebec · Accommodation · Tours · Where to Stay

Quebec City - Wikipedia, the free encyclopedia



**Reward: revenue** 

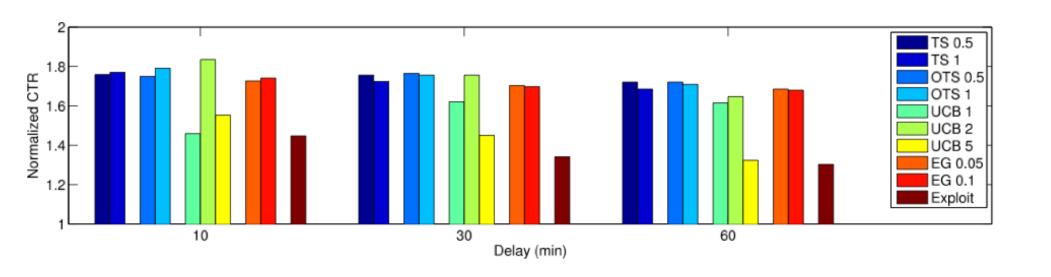
# Limitation of UCB in Online Advertising

- How to take advantage of prior information to avoid unnecessary exploration
- How to handle long delay of reward after taking an action
- How to enable complex models

# Thompson Sampling

- Old heuristic: "probability matching" (1933)
  - $Pr(a|\mathbf{x}) = Pr(a \text{ is optimal for } \mathbf{x} \mid prior, data)$
- Highly effective in practice [Scott'10] [CL'11]
- Inspired lots of theoretical study in last 2 years
  - Non-contextual bandits [Agrawal, Goyal, Kaufmann, ...]
  - Linear bandit [Agrawal & Goyal'13]
  - Generalized Thompson Sampling [L'13]
  - Bayes risk analyses [Russo & Van Roy]

# Thompson Sampling for Advertising [CL'12]



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# Model-agnostic Algorithms

- EXP4 [Auer et al'95] [BL<u>L</u>RS'11]
  - Optimal  $O(\sqrt{T})$  regret bound
  - Works even when contexts and rewards are generated by adversarial
  - Computationally expensive in general
- ILOVETOCONBANDITS [AHKLLS'14]
  - Optimal  $O(\sqrt{T})$  regret bound
  - Computationally efficient
  - Promising empirical results

# Outline

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# Policy Evaluation

Assume stochastic bandit:  $\mathbf{x} \sim \nu$ ,  $r_a \sim \nu$  ( $\cdot | \mathbf{x}, a$ )

Given a policy  $\pi: \mathbf{X} \to a$ , want to estimate its value:  $V(\pi) = \mathbf{E}_{\mathbf{v}}[r_{\pi(\mathbf{X})}]$ 

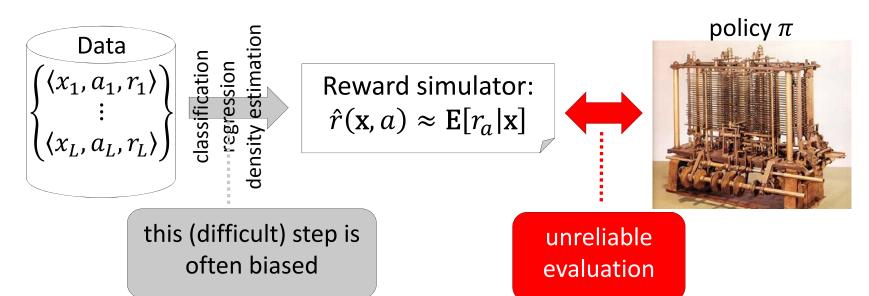
### Online evaluation

- Run  $\pi$  on live users and average observed rewards (as in A/B tests)
- Reliable but expensive

### Offline evaluation

- Estimate  $V(\pi)$  from historical data set  $D = \{(\mathbf{x}, a, r_a)\}$
- Fast and cheap (e.g., benchmark data sets for supervised learning)
- Counterfactuality of rewards: no information to evaluate  $\pi$  if  $\pi(x) \neq a$

# Common Approach in Practice



### In contrast, our approach

- avoids explicit user modeling → simple
- gives unbiased evaluation results → reliable

# Our Approach: Unbiased Offline Evaluation

Randomized data collection: at step t,

- Observe current context x
- Randomly chooses  $a \in A$  according to  $(p_1, p_2, ..., p_K)$  and receives  $r_a$ End result: "exploration data"  $D = \{(\mathbf{x}, a, p_a, r_a)\}$

### Key properties:

- Unbiasedness:  $\mathbf{E}_D\left[\frac{1}{|D|}\sum_{(\mathbf{X},a,p_a,r_a)\in D}\frac{r_a\cdot\mathbf{1}(\pi(\mathbf{X})=a)}{p_a}\right]=V(\pi)$
- Estimation error =  $O\left(\frac{1}{\sqrt{|D|}}\right)$

Related to causal inference (Neyman-Rubin) and off-policy learning [Precup & co]

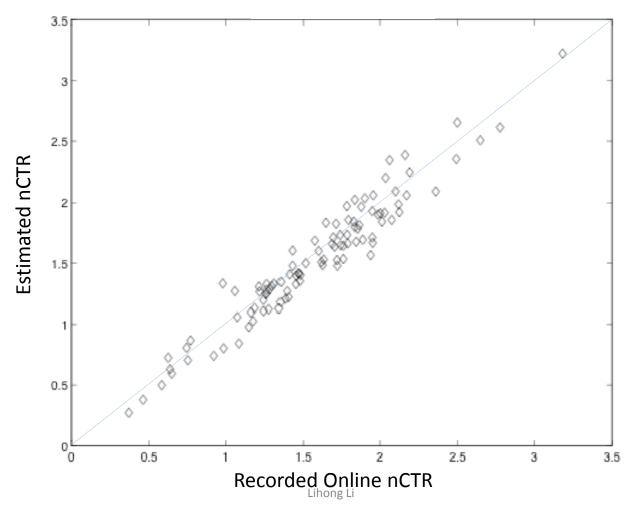
# Case 1: News Recommendation [LCLW'11]

- Experiments run in 2009
- ullet Fixed an article-selection policy  $\pi$

- Run  $\pi$  on live users to measure online click rate
  - The ground truth
- Use exploration data to evaluate  $\pi$ 's click rate
  - The offline estimate

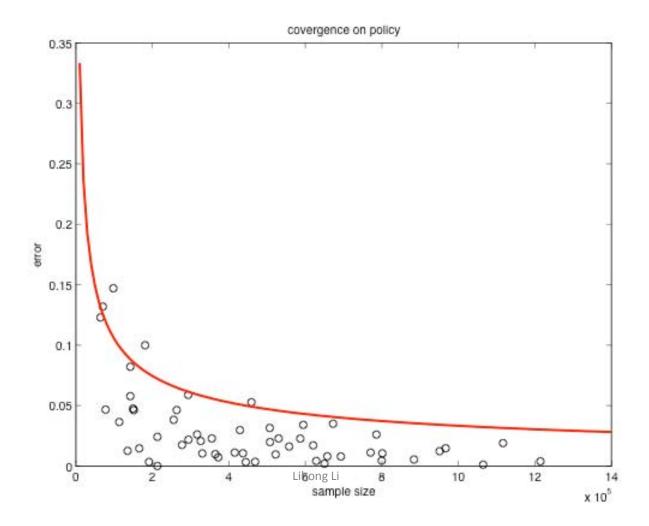


# Unbiasedness



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# **Estimation Error**



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# Case 2: Spelling Correction of Bing





### **What Speller does:**

- Corrects typos
- May produce multiple candidates (with search results blended later)

### **Objective:**

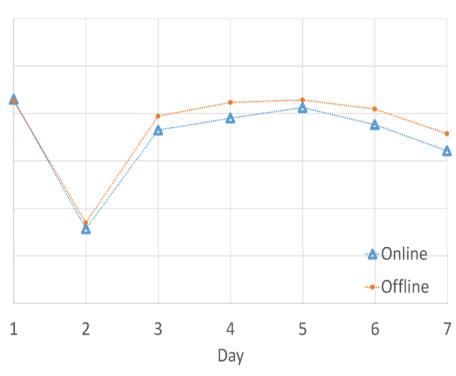
To optimize pre-defined click metrics

# Accuracy of Offline Evaluator [LCKG'14]

### Position-specific click-through rate

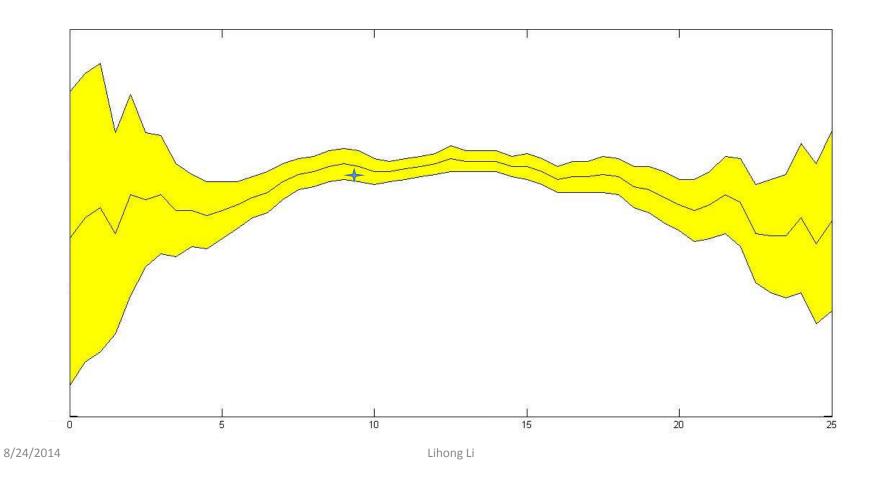
# 1 2 3 4 5 6 7 8 Position A Online Offline

### Daily click-through rate



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# Quantifying Uncertainty in Offline Evaluation



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# Case 3: Web Search Ranking



### contextual bandit



37,000 RESULTS

Any time ▼

### Multi-armed bandit - Wikipedia, the free encyclopedia

en.wikipedia.org/wiki/Contextual\_bandit\_algorithm \*

In probability theory, the multi-armed **bandit** problem (sometimes called the K - or N - armed **bandit** problem) is the problem a gambler faces at a row of slot machines ... Empirical motivation · The multi-armed ... · Variations · **Bandit** strategies

### Contextual Bandits « Machine Learning (Theory)

hunch.net/?p=298 ▼

With rich **contextual** information my first instinct is to restructure the presentation strategy to get back into a supervised learning framework.

### PDFI A Contextual-Bandit Approach to Personalized News ...

www.research.rutgers.edu/~lihong/pub/Li10Contextual.pdf

A **Contextual-Bandit** Approach to Personalized News Article Recommendation Lihong Li†, WeiChu†, †Yahoo! Labs lihong,chuwei@yahoo-inc.com John Langford‡

### [PDF] Contextual Multi-Armed Bandits - Department of ...

www.cs.toronto.edu/~tl/papers/context-aistats.pdf

### Search as a bandit:

- Context: query
- Action: ranked list
- Reward: search success-or-not

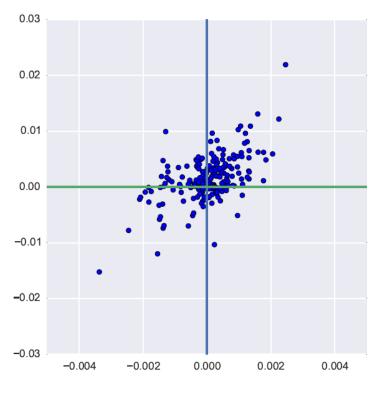
### Challenges:

- Exponentially many actions
  - → large estimation variance
- Collecting enough randomized data can be too expensive

# Trading off Bias and Variance [LKZ'14]

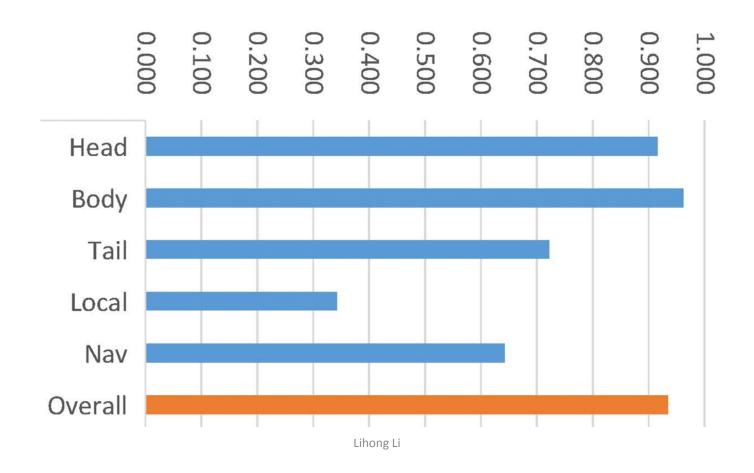
- Use natural exploration (uncontrolled diversity) of Bing to simulate randomized data collection
  - Nearly unbiased [SL<u>L</u>K'11]
  - $\triangleright$  Can use unlimited amount of data  $\rightarrow$  lower variance
- Use approximate matching of actions
  - ➤ May introduce some amount of bias
  - Can dramatically reduce variance

# Predicting Success of New Ranking Function



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# Metric Correlation based on Query Segments



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# Advanced Offline Evaluation Techniques

- Doubly robust estimation [DL<u>L</u>'11]
- Extends to evaluate learning algorithms (e.g., LinUCB) [DEL<u>L</u>'12]
  - With adaptive importance sampling

Increasingly popular at industrial leaders.

# Outline

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## Conclusions

- Contextual bandits as a natural and versatile model
  - Better decision making → causality
  - Rich information enables better user understanding & decision making
- Additional challenges not seen in traditional bandits
  - Delayed rewards, decision making with constraints, ...
  - Dueling bandit [Yue & co]
  - Gang of bandits [Cesa-bianchi & co]
  - ...
- Large amount of data makes offline evaluation feasible
  - Can validate offline evaluation precision by experiments
  - What is the optimal estimator? [LMS'14]
- Next big step: full RL (non-myopic decision making)